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The study of transient phenomena in selective circuits is of great practical interest. It is sufficient to point out that a clarification of the effect of pulse disturbances on radio reception is essentially connected with the study of transient phenomena. An examination of transient phenomena also yields information on the proper selection of circuit parameters for pulse transmitting and receiving circuits.

Finally, when a single pulse is applied to the input of a four-terminal network, its properties are characterized as much by transient functions as they are by its frequency and phase characteristics. In this sense, transient functions have a well-known independent usefulness.

Many works have been devoted to the general aspects of transient phenomena in electrical circuits, but transient phenomena in circuits intended specifically for radio engineering work have been studied to a lesser degree. It is true that for the solution of problems concerned with transient phenomena in such circuits, there exists the complete apparatus of operational calculus and circuit integrals which has been fully developed by a number of investigators starting with Heaviside. However, in general, the use of this apparatus presents great calculation difficulties which make it impossible to carry the calculations to their logical conclusion and necessitates the omission of a number of factors.

The specific properties of selective circuits permit one to devise, on the basis of classical methods of solving nonstationary problems, a special approximate method which quickly leads to the objective and frees one from the necessity of making omissions for each concrete calculation.

In practice, it is not necessary to know the fine details of the transient phenomena taking place in selective circuits. Of primary interest is the determination of the envelope and phase of the voltage at the output of the four-terminal network. The high-frequency component of this envelope can always be represented by harmonic functions with a frequency equal to, or approximating, the resonant frequency. With a small μ , the change in the envelope in one period of the resonant frequency will be small; therefore, the initial phase of the high frequency component is of no practical interest. Using the terminology characterizing van der Pol's method, it can be said that the envelope and the phase will be slowly varying functions of time.

The calculations can be simplified considerably if we introduce the concept of the complex, slowly varying, amplitude. Its determination immediately solves the problem regarding the law of variation not only of the envelope but of the current phase in the process of establishment. The introduction of the transient complex amplitude permits a simple solution of the problem of transient phenomena during the detuning of the frequency of the external effect relative to the resonant frequency, by manipulating the frequency, etc. When we use the word "envelope," we shall, in general, assume it to mean the complex amplitude.

The basic theorems for calculating transient envelopes on the output side of selective circuits, are given below.

Dubamel's Theorem for Envelopes

When a single pulse is applied to the input of a selective system, the transient process is expressed on the output side as a harmonic voltage of frequency equal to the resonant frequency and with a slowly varying amplitude. This voltage may be written in the form

$$a(t) = A(t) \sin \omega_0(t) \quad (1)$$

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In accordance with Duhamel's theorem, when a voltage $e(t)$ is present at the input, a voltage $u(t)$ will appear at the output which can be expressed as follows:

$$u(t) = a(0)e(t) + \int_0^t a^1(\tau)e(t-\tau)d\tau \quad (2)$$

Let us designate the slowly varying amplitude and phase of the output voltage by $B(t)$ and $\Psi(t)$, respectively. Then the complex amplitude of the output voltage will be

$$\vec{B}(t) = B(t)e^{j\Psi(t)}. \quad (3)$$

From previous equations, it now follows that the complex amplitude of the output voltage is expressed by

$$\vec{B}(t) = \frac{\omega_0}{2} \int_0^t A(\tau)E(t-\tau)e^{-jA\omega_0\tau}d\tau \quad (4)$$

This formula, therefore, is the Duhamel integral for the envelopes and will play a fundamental role in the calculations of transient phenomena.

A particularly important case arises when the envelope of the input voltage has the form of a single pulse, i.e.,

$$E(t) = 1.$$

Let $B_1(t)$ designate the complex amplitude at the output for this case to distinguish it from all other cases.

By a series of operations, we finally obtain the formula

$$A(t) = \frac{2}{\omega_0} \frac{d}{dt} B_1(t); \quad (5)$$

this formula is used in the succeeding section to derive the equations for calculating $A(t)$. For the time being we may deduce the following results from formula (5): Inasmuch as $A(t)$ is a slowly varying function of time, it follows that $A(t)$ is a value of the first order of smallness, i.e., proportional to μ . Thus, from formula (5) we may reach the very general conclusion that $A(t)$ is proportional to the pass band and inversely proportional to the resonant frequency.

Calculating Envelopes for Single Pulses

In calculating $A(t)$ we shall again attempt to use the approximate methods characteristic for van der Pol's methods. With this in mind, let us use the established relationship between van der Pol's "abridged" differential equations and the symbolic equations [17].

Let us assume that for a selective four-terminal network, in accordance with the usual rules for AC circuits with a frequency ω , the transfer factor takes on a complex form:

$$k = k(j\omega).$$

If we look at $j\omega$ as a differential operator $j\omega = \frac{d}{dt}$, then the instantaneous voltages at the input and output will be connected by the following symbolic equation:

$$u(t) = k(j\omega)e(t) \quad (6)$$

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On the basis of the reasoning employed in the author's previous work [1], it can be proved that the complex, slowly varying, amplitudes of the voltages on the input ($E(t)e^{j\Delta\omega t}$) and output ($B(t)e^{j\Delta\omega t}$) will be connected by a symbolic equation, equivalent to the van der Pol's abridged equation, in the following form:

$$\vec{B}(t)e^{j\Delta\omega t} = k(j\Omega)E(t)e^{j\Delta\omega t} \quad (7)$$

In this equation, $k(j\Omega)$ denotes the so-called "abridged" expression for the transfer factor, worked out for small detunings close to the resonant frequency.

Applying the theorem of displacement [3] to equation (7), we obtain

$$\vec{B}(t) = k(j\Omega + j\Delta\omega)E(t). \quad (8)$$

Equation (8) makes it possible to calculate $B(t)$ without resorting to Duhamel's integral method. For this purpose, we may use the operational calculus methods which boil down to a Laplace transformation of the function $E(t)$, and then to a calculation of $B(t)$ from Bromwich's integral. In the detached cases when $E(t)$ equals 1, it is possible to use Heaviside's expansion formula.

We shall not develop here the application of these methods to a solution of equation (8), since in concrete problems the Duhamel integral method is less difficult and yields the desired results quicker. Equation (8) holds our attention in another respect. The difference between equation (8) and Duhamel's integral (4) is essential. This difference is based on the fact that in equation (4) the properties of the four-terminal network are given by the envelope $A(t)$ while in equation (8), they are characterized by the transfer factor $k(j\Omega)$. Therefore, by comparing equations (8) and (4), it is possible to find the symbolic equation for determining $A(t)$ through $k(j\Omega)$.

For the case when there is no detuning ($\Delta\omega = 0$) and a unit amplitude at the input ($E = 1$) we obtain, from formula (8),

$$\vec{B}_1(t) = k(j\Omega) \cdot 1. \quad (9)$$

On the other hand, from Duhamel's integral for the same case we had already obtained equation (5) in which $A(t)$ is determined through $B_1(t)$. Rewriting equation (5) in symbolic form and substituting 1 for the derivative the differential operator $\frac{d}{dt} = j\Omega$

$$A(t) = \frac{2}{\omega_0} j\Omega B_1(t). \quad (5)$$

If we now substitute equation (9) in equation (5) we shall obtain the following symbolic equation for determining $A(t)$:

$$A(t) = \frac{2}{\omega_0} j\Omega k(j\Omega) \cdot 1. \quad (10)$$

Equation (10) can be solved in a number of different ways depending on which of them is most expedient for a given case (Heaviside's power series, etc.).

The most general procedure is to find a solution from the integral representation of the function $A(t)$. This leads to the following equation:

$$A(t) = \frac{1}{\pi\omega_0} \int_{-\infty}^{\infty} k(j\Omega) e^{j\Omega t} d\Omega \quad (11)$$

Equation (11) represents $A(t)$ in the form of an infinite number of harmonic components of the form $e^{j\Omega t}$ with a spectral density of $\frac{1}{\pi\omega_0} k(j\Omega)$. The calculation of $A(t)$ usually leads to the conclusion that, on basis of Jordan's lemma, the integral of equation (11) is replaced by a definite integral. In this connection, it

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is necessary to keep in mind that the limits of integration must encompass all specific points of the function under the integral, $k(j\Omega)$. In such a case, the integral of formula (11) is equal to the sum of the "deductions" of function $k(j\Omega)$ and a solution for $A(t)$ is obtained in final form.

Equations (4) and (10) and their important particular cases -- equation (11) among others -- comprise the basic results of the theory developed here for calculating the envelopes for transient phenomena in selective four-terminal networks.

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